Random Graphs
Exercise Sheet 2

Question 1. Consider the model of a random bipartite graph $G(n, n, p)$ on two equal size vertex classes of size n, where each edge is included in the graph independently with probability p.

Show that the function $\hat{p}(n) = \frac{1}{n}$ is a threshold function for the event that the graph contains a 4-cycle (that is, a complete bipartite graph $K_{2,2}$).

Question 2. Determine a threshold for the property of having diameter at most 2. Is the threshold sharp?

Question 3. Let k be a fixed integer. Show that if $p = \omega\left(\frac{\log(n)}{n}\right)$ then the expected number of independent sets of size $\frac{n}{k}$ in $G_{n,p}$ tends to 0. Show that if $p = o(\frac{1}{n})$ then the expected number will tend to infinity.

Question 4. Let g be a fixed integer. Show that if $p = o(\frac{1}{n})$ then the expected number of cycles of length at most g in $G_{n,p}$ tends to 0.

Show that if $p = n^{\frac{1}{2g}-1}$ then the expected number of cycles of length at most g is $o(n)$.

Question 5. Let g and k be fixed integers. Show that there exists a graph with $g(G) \geq g$ and $\chi(G) \geq k$.

Question 6. Let $p = \frac{2+\varepsilon}{n}$. Show that with high probability $G_{n,p}$ is non-planar.

(Hint : Find a subgraph with large girth)